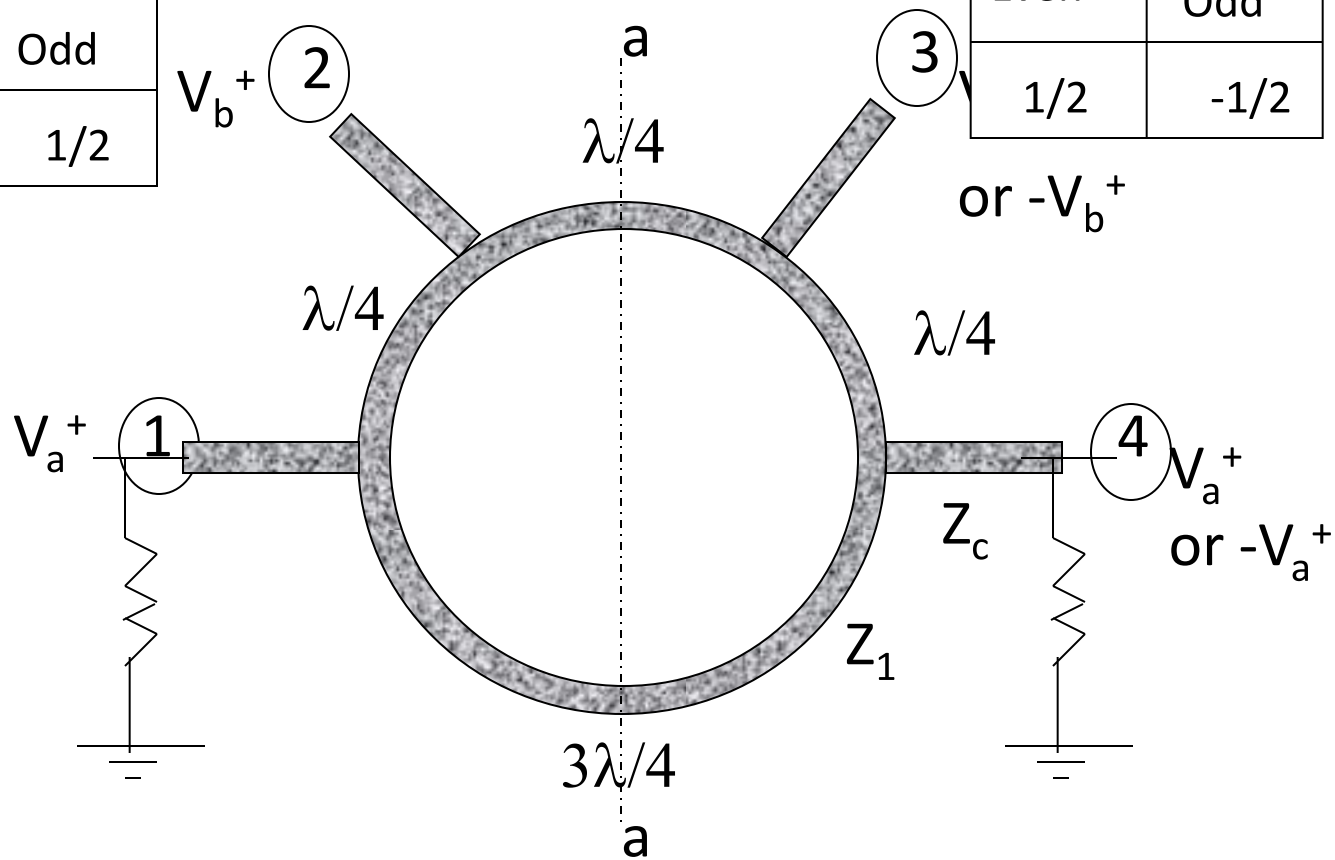
UNIT-2

Microwave Engineering

Microstrip Hybrid Ring

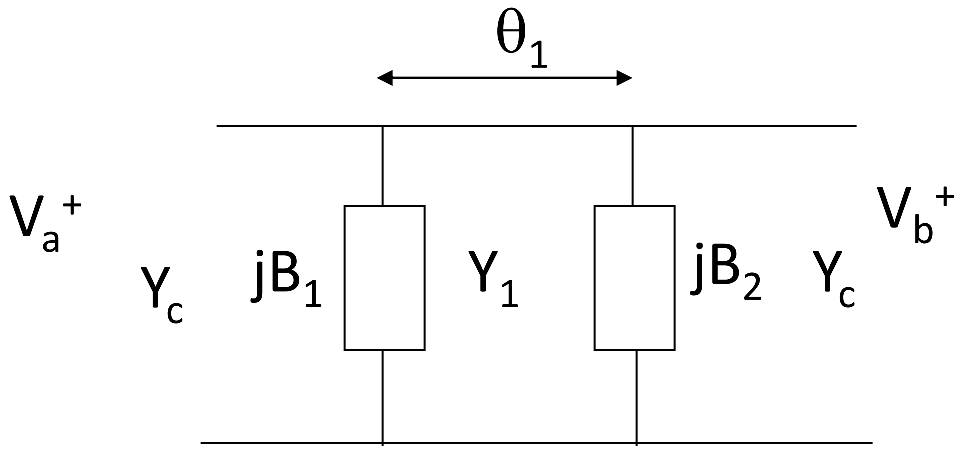
Even	Odd
1/2	1/2

Even	Odd
1/2	-1/2



The hybrid ring (rat-race)

Ports 1 and 3 are uncoupled, ports 2 and 4 are uncoupled



Equivalent circuit for
one half of Hybrid ring

For even excitation :

$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11}^{oc} & S_{12}^{oc} \\ S_{21}^{oc} & S_{22}^{oc} \end{bmatrix} \begin{bmatrix} V_a^+ \\ V_b^+ \end{bmatrix} \quad , \quad \begin{bmatrix} V_4^- \\ V_3^- \end{bmatrix} = \begin{bmatrix} S_{11}^{oc} & S_{12}^{oc} \\ S_{21}^{0c} & S_{22}^{oc} \end{bmatrix} \begin{bmatrix} V_a^+ \\ V_b^+ \end{bmatrix}$$

For odd excitation :

$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11}^{sc} & S_{12}^{sc} \\ S_{21}^{sc} & S_{22}^{sc} \end{bmatrix} \begin{bmatrix} V_a^+ \\ V_b^+ \end{bmatrix} \quad , \quad \begin{bmatrix} V_4^- \\ V_3^- \end{bmatrix} = \begin{bmatrix} S_{11}^{sc} & S_{12}^{sc} \\ S_{21}^{sc} & S_{22}^{sc} \end{bmatrix} \begin{bmatrix} -V_a^+ \\ -V_b^+ \end{bmatrix}$$

Use superposition $V_1^+ = 2V_a^+, V_2^+ = 2V_b^+$

$$V_3^+ = V_4^+ = 0$$

$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \frac{1}{2}\begin{bmatrix} S_{11}^{oc} + S_{11}^{sc} & S_{12}^{oc} + S_{12}^{sc} \\ S_{21}^{oc} + S_{21}^{sc} & S_{22}^{oc} + S_{22}^{sc} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

$$\begin{bmatrix} V_4^- \\ V_3^- \end{bmatrix} = \frac{1}{2}\begin{bmatrix} S_{11}^{oc} - S_{11}^{sc} & S_{12}^{oc} - S_{12}^{sc} \\ S_{21}^{oc} - S_{21}^{sc} & S_{22}^{oc} - S_{22}^{sc} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

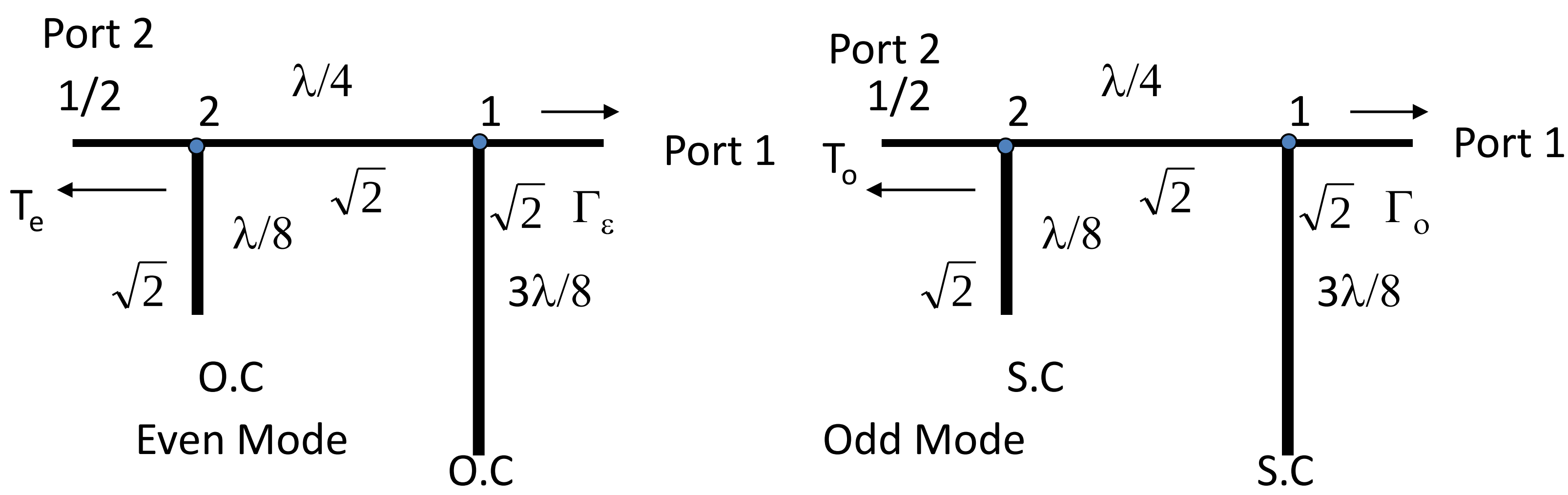
$$S_{11} = \frac{1}{2}(S_{11}^{oc} + S_{11}^{sc}) = \frac{1}{2}(\Gamma_e + \Gamma_o) \quad , \quad S_{12} = \frac{1}{2}(S_{12}^{oc} + S_{12}^{sc}) = \frac{1}{2}(T_e + T_o)$$

$$S_{21} = \frac{1}{2}(S_{12}^{oc} + S_{12}^{sc}) \quad , \quad S_{22} = \frac{1}{2}(S_{22}^{oc} + S_{22}^{sc})$$

$$S_{31} = \frac{1}{2}(S_{12}^{oc} - S_{12}^{sc}) = \frac{1}{2}(\Gamma_e - \Gamma_o) \quad , \quad S_{32} = \frac{1}{2}(S_{22}^{oc} - S_{22}^{sc}) = \frac{1}{2}(T_e - T_o)$$

$$S_{41} = \frac{1}{2}(S_{11}^{oc} - S_{11}^{sc}) \quad , \quad S_{42} = \frac{1}{2}(S_{12}^{oc} - S_{12}^{sc})$$

$$S_{44} = S_{11}, \quad S_{33} = S_{22}, \quad S_{34} = S_{21}$$



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_e = \begin{bmatrix} -1 & j\sqrt{2} \\ j\sqrt{2} & 1 \end{bmatrix} \quad , \quad \begin{bmatrix} A & B \\ C & D \end{bmatrix}_o = \begin{bmatrix} 1 & j\sqrt{2} \\ j\sqrt{2} & -1 \end{bmatrix}$$

$$\Gamma_e = \frac{j}{\sqrt{2}} \quad , \; T_e = \frac{j}{\sqrt{2}} \quad , \; \Gamma_o = \frac{-j}{\sqrt{2}} \quad , \; T_o = \frac{-j}{\sqrt{2}}$$

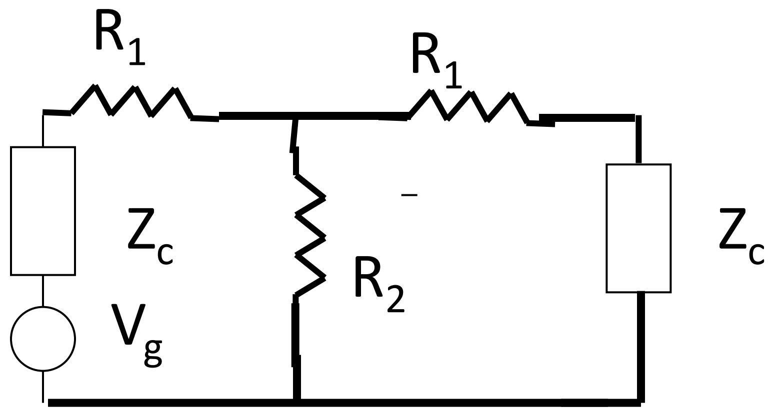
$S_{11} = o$ i.e. port1 is matched.

$$S_{21} = \frac{-j}{\sqrt{2}} \quad , \qquad S_{31} = \frac{j}{\sqrt{2}} \quad , \; S_{31} = 0$$

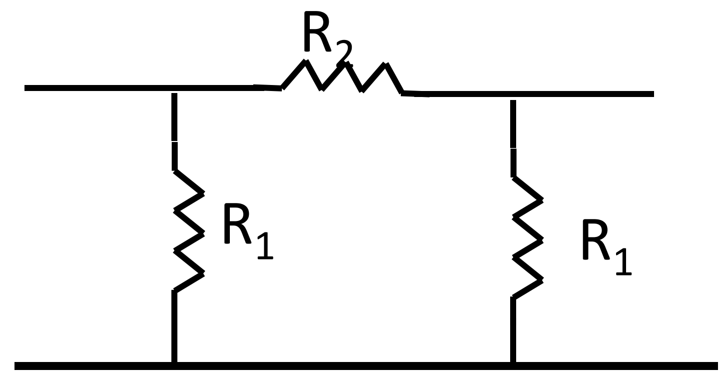
$$[S] = \frac{-j}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 0 & -1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ -1 & 0 & 1 & 0 \end{bmatrix}$$

Passive Microwave Devices

Attenuators



T section



π section

$$R_{in} = R_1 + \frac{R_2 (R_1 + Z_c)}{R_1 + R_2 + Z_c}$$

For $R_{in} = Z_c \Rightarrow R_1 (R_1 + 2R_2) = Z_c^2$

$$V_{TH} = \left(\frac{R_2}{R_1 + R_2 + Z_c} \right) V_g$$

The power delivered to the load is :

$$P_L = \frac{1}{2}\left|\frac{V_{TH}}{2Z_c}\right|^2 Z_c = \left(\frac{R_2}{R_1 + R_2 + Z_c}\right)^2 \frac{\left|V_g\right|^2}{8Z_c} = K^2 \frac{\left|V_g\right|^2}{8Z_c}$$

$$K^2 = \left(\frac{R_2}{R_1 + R_2 + Z_c}\right)^2$$ Power attenuation, $\mathrm{R}_{\mathrm{in}} = Z_c \Rightarrow$

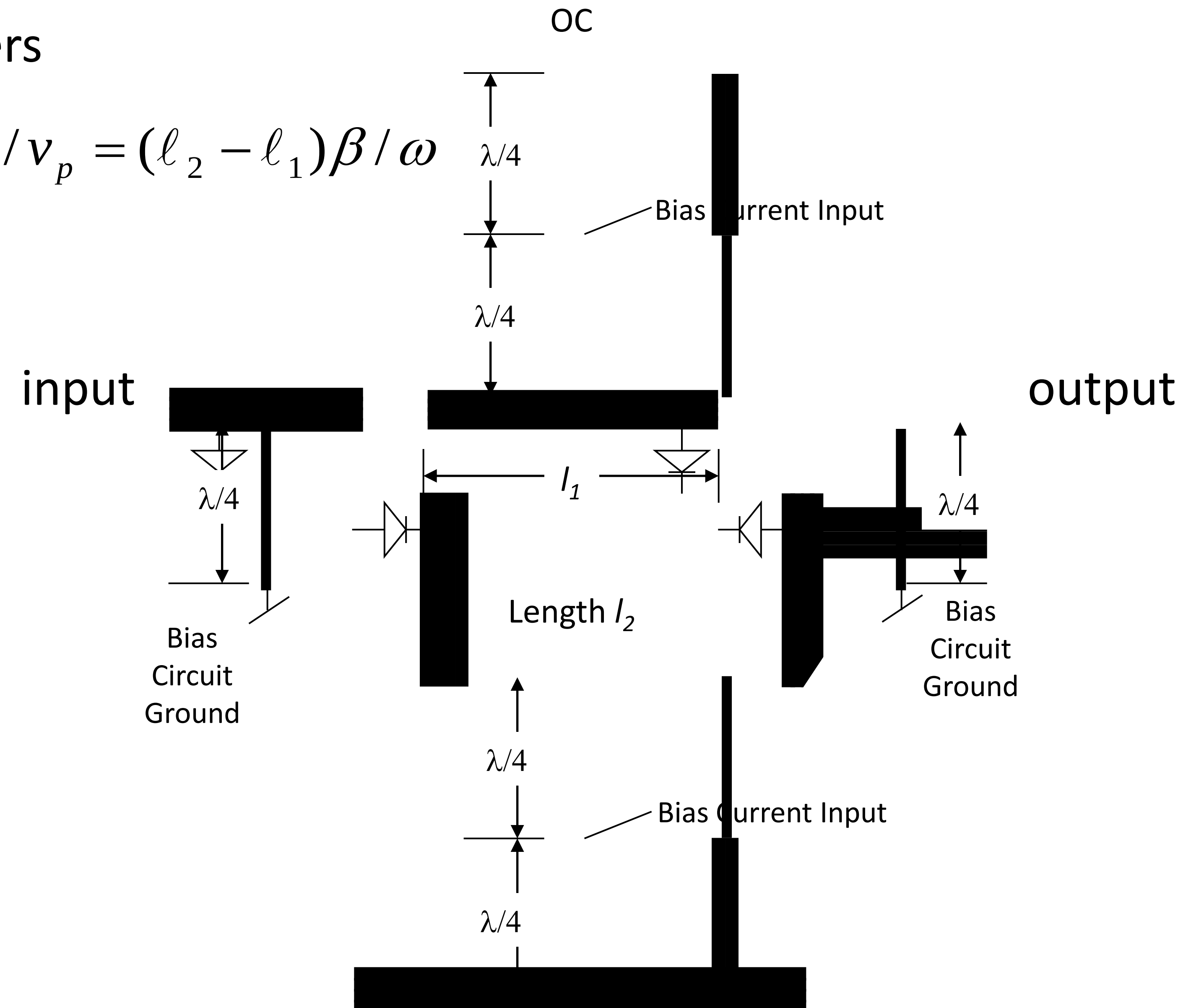
$$\mathrm{R}_1 = \frac{1-K}{1+K} Z_c \quad , \quad \mathrm{R}_2 = \frac{2K}{1-K^2} Z_c$$

For 3-dB attenuator $\mathrm{R}_1 = 8.58\Omega$, and $\mathrm{R}_2 = 141.4\Omega$.

PIN diode can be used in parallel with R_2 in the π-section configuration to switch it in and out of the circuit

Phase shifters

$$\Delta\tau = (\ell_2 - \ell_1)/v_p = (\ell_2 - \ell_1)\beta/\omega$$



Incremental-line-type phase shifter